

# The Standard NAFEMS Benchmark Tests for CEETRON Solve

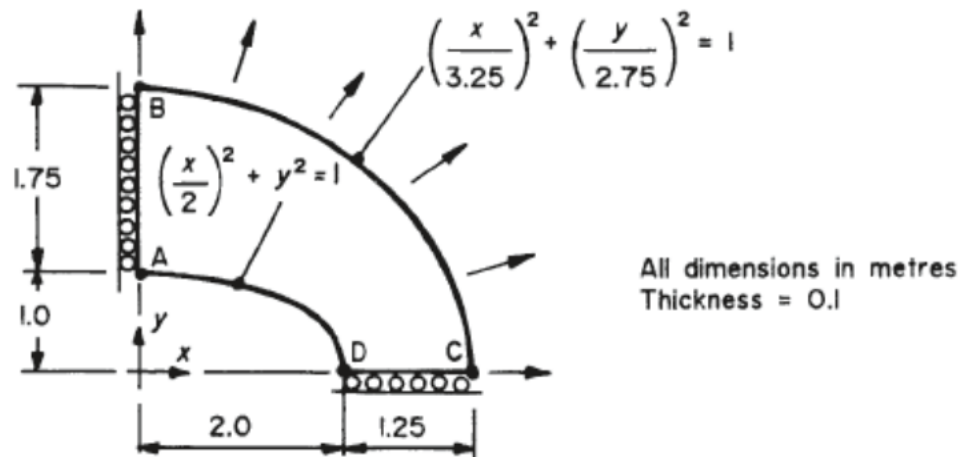
This document contains NAFEMS benchmark tests for CEETRON Solve version 2.10.0. Each test case provides a detailed overview of the problem and results compared between theoretical expectations, CEETRON Solve results, and MSC Nastran results.

# **Linear Elastic**

# Elliptic Membrane

## Problem Description:

Find the tangential edge stress at D (refer below figure) of elliptical membrane which is uniformly loaded with outward pressure



## Reference:

Test LE1 from NAFEMS Publication NNB, Rev. 3, NAFEMS Linear Benchmarks, 5 Oct 1990

## Modeling Techniques Used:

Elliptical membrane is modeled using plane stress elements quadrilaterals.  
Linear Static Analysis

## Loading:

Uniform outward pressure of 10MPa at outer edge BC, Inner curved edge AD unloaded.

## Boundary Condition:

Edge AB, symmetry about Y axis, e.g. zero x displacement  
Edge CD, symmetry about X axis, e.g. zero y displacement.

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  Mpa and  $\nu = 0.3$

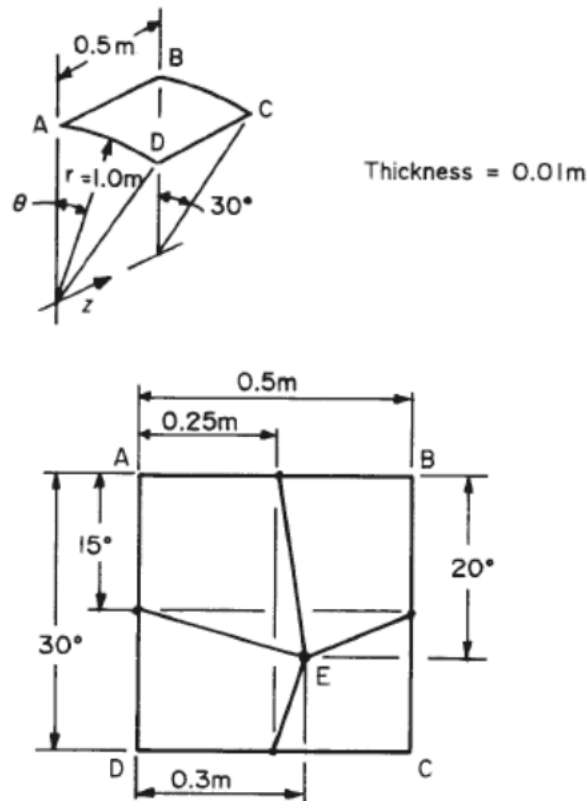
## RESULTS:

Theory	CEETRON Solve	MSC Nastran
92700000.0	94812752.0	91160000.0

# Cylindrical Shell Patch Test

## Problem Description:

Find the outer surface tangential stress at E (refer below figure) of cylindrical shell with uniform normal edge moment, on DC of  $1.0 \text{ kNm/m}$



## Reference:

Test LE2 from NAFEMS report TSBM, Publication NNB, Rev. 3, NAFEMS Linear Benchmarks, 5 Oct 1990

## Modeling Techniques Used:

Cylindrical shell is modeled using plane stress quadrilateral elements.  
Linear Static Analysis

## Loading:

Uniform normal edge moment, on DC, of  $1.0 \text{ kNm/m}$

## Boundary Condition:

Edge AB, all translations and rotations zero  
Edge AD, BC are symmetric about r-theta plane, e.g. Z translations and normal rotations all zero.

## Material Properties:

Isotropic,  $E = 210 \times 10^3 \text{ MPa}$  and  $\nu = 0.3$

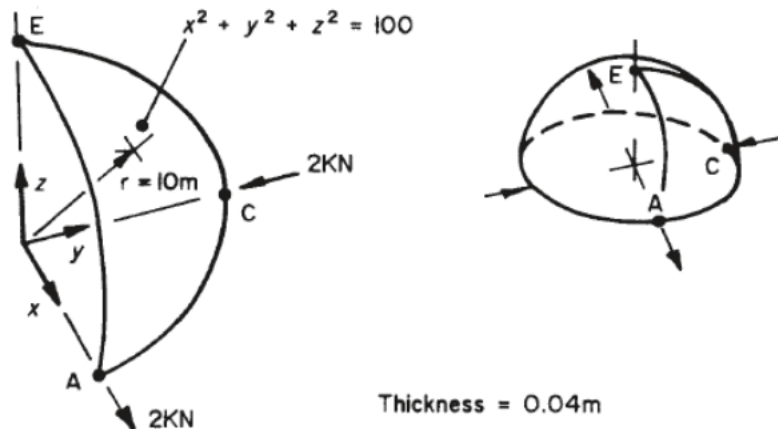
**RESULTS:**

Theory	CEETRON Solve	MSC Nastran
60000000.0	65775087.0	57300000.0

# Hemisphere-Point Loads

## Problem Description:

Find the displacement in x- direction of point A of hemisphere shown below



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE3. Glasgow: NAFEMS, Rev. 3, 1990

## Modeling Techniques Used:

Hemisphere shell is modeled using plane stress quadrilateral elements.  
Linear Static Analysis

## Loading:

Uniform normal pressure of  $1\text{ MPa}$  on the upper surface of the plate

## Boundary Condition:

Point E, zero  $z$  displacement  
Edge AE, symmetry about  $zx$  plane; e.g zero  $y$  displacement, zero normal rotation  
Edge CE, symmetry about  $yz$  plane, e.g zero  $x$  displacement, zero normal rotation

## Material Properties:

Isotropic,  $E = 68.25 \times 10^3\text{ Mpa}$ ,  $\nu = 0.3$

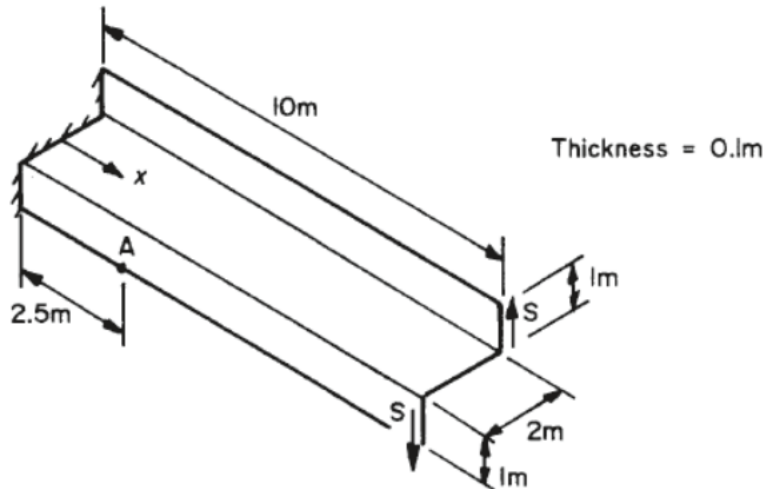
## RESULTS:

Theory	CEETRON Solve	MSC Nastran
0.185	0.176	0.179

# Z-section Cantilever

## Problem Description:

A z-section cantilever beam is subjected to a torsion load. Find the axial stress (X-X) at mid-surface at Point A shown below



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE5. Glasgow: NAFEMS, Rev. 3, 1990

## Modeling Techniques Used:

Z shaped cantilever beam is modeled using plane stress quadrilateral elements  
Linear Static Analysis

## Loading:

Torque of 1.2 MNm applied at end  $x = 10$  by two uniformly distributed. Edge shears,  $S = 0.6$  at each flange.

## Boundary Condition:

At edge  $x = 0$ , all displacements are zero.

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  MPa,  $\nu = 0.3$

## RESULTS:

Theory	CEETRON Solve	MSC Nastran
-108000000.0	-104137508.0	-103000000.0

# Skew Plate Normal Pressure

## Problem Description:

A skew plate is subjected to uniform normal pressure in the vertical z- direction. 4 node quadrilateral element is used. Find the maximum principal at plate center on bottom section.

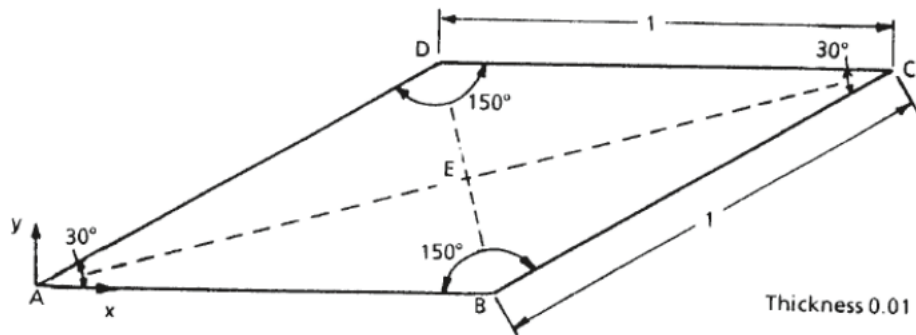


Figure 6.5-1 Skew Plate

## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE6. Glasgow: NAFEMS, Rev. 3, 1990.

## Modeling Techniques Used:

Linear Static Analysis

## Loading:

Normal pressure of -0.7KPa in vertical Z-direction

## Boundary Condition:

Simple supports (no Z-displacement) for all edges AB, BC, CD, DA

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  MPa,  $\nu = 0.3$

## RESULTS:

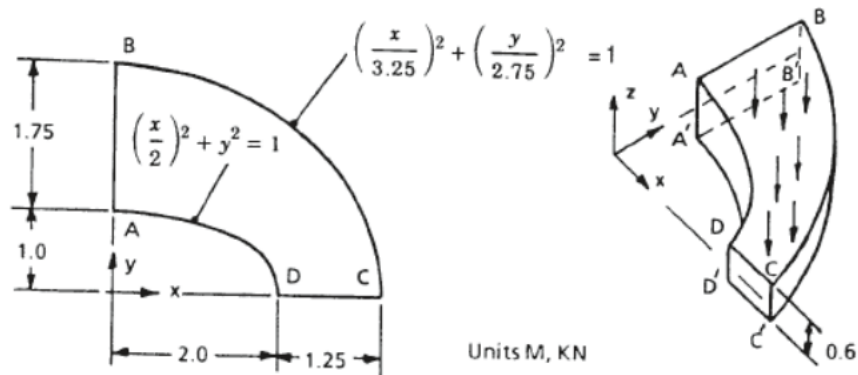
Theory	CEETRON Solve	MSC Nastran
802000.0	703654.962	768000.0



# Thick Plate Pressure

## Problem Description:

A elliptical thick plate is subjected to uniform normal pressure on the upper surface of the plate. Find the direct Stress (Y-Y) at Point D shown below



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE10. Glasgow: NAFEMS, Rev. 3, 1990

## Modeling Techniques Used:

Thick elliptical plate is modeled using solid hexahedra elements. Linear Static Analysis

## Loading:

Uniform normal pressure of 1 MPa on the upper surface of the Plate

## Boundary Condition:

Face DCD'C' zero y-displacement

Face ABA'B' zero x- displacement

Face BCB'C' x and y displacements fixed, z displacements fixed along mid-plane

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  MPa,  $\nu = 0.3$

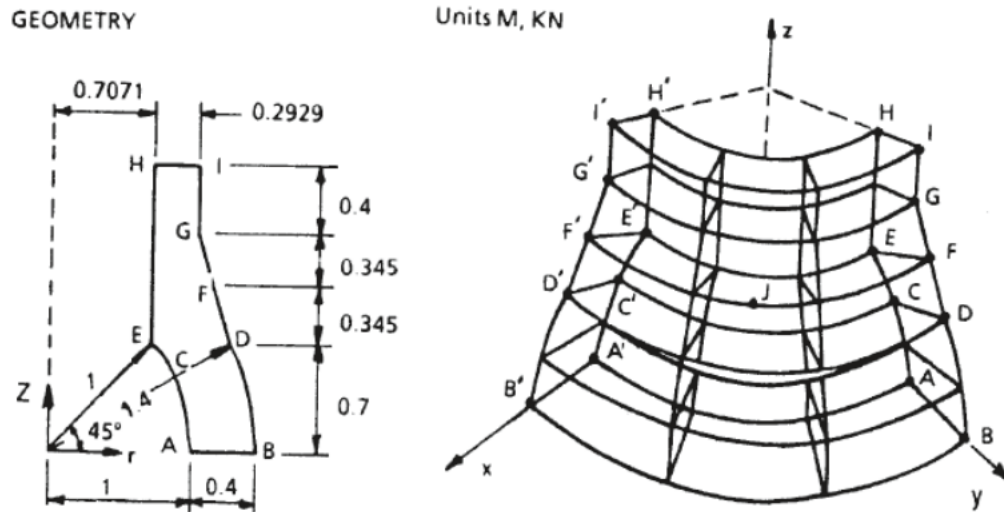
## RESULTS:

Theory	CEETRON Solve	MSC Nastran
-5380000.0	-5642920.5	-6000000.0

# Solid Cylinder/ Taper/ Sphere-Temperature

## Problem Description:

A solid spherical taper cylinder is subjected temperature loading. Find the direct stress (Z-Z) at Point A



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE11. Glasgow: NAFEMS, Rev. 3, 1990.

## Modeling Techniques Used:

Solid cylinder is modeled using solid elements hexahedra. Linear Static Analysis

## Loading:

Linear temperature gradient in the radial and axial direction  $T (^{\circ}\text{C}) = (x^2 + y^2)^{1/2} + z$

## Boundary Condition:

Symmetry on x-z plane i.e., zero y- displacement,  
Symmetry on y-z plane i.e., zero x- displacement,  
Face on xy plane zero z- displacement,  
Face HH'I' zero z- displacement

## Material Properties:

Isotropic,  $E = 210 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ ,  $\alpha = 0.00023 \text{ }^{\circ}\text{C}$

## RESULTS:

Theory	CEETRON Solve	MSC Nastran
-105000000.0	-93465800.0	-99477000.0

# Free Vibration

# Cantilever with Off-Centre Point Masses

## Problem Description:

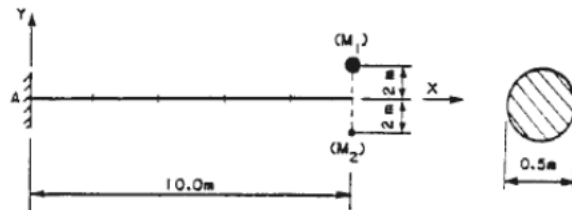
Calculation of natural frequencies (Hz) of the first 6 modes of a cantilever beam with offset masses at the free end.

### Geometry and Mesh

Exact beam: 5 elements along cantilever

$$M_1 = 10000 \text{ kg} \quad (\text{along X, Y, Z})$$

$$M_2 = 1000 \text{ kg} \quad (\text{along X, Y, Z})$$



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV4. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

Simple cantilever beam model.  
BEAM elemnts.  
Coupling between bending and torsion  
Close eigenvalues  
Inertial axis non-coincident with flexibility axis.

## Boundary Condition:

$$x = y = z = R_x = R_y = R_z = 0 \text{ at A}$$

## Material Properties:

$$E = 200 \times 10^3 \text{ MPa}, \nu = 0.3, \text{ density} = 8000 \text{ kg/m}^3$$

## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	1.723	1.723	1.723
2	1.727	1.727	1.727
3	7.413	7.424	7.45
4	9.972	9.972	9.975
5	18.155	18.162	18.205
6	26.957	26.973	27.001

# Deep Simply-supported Beam

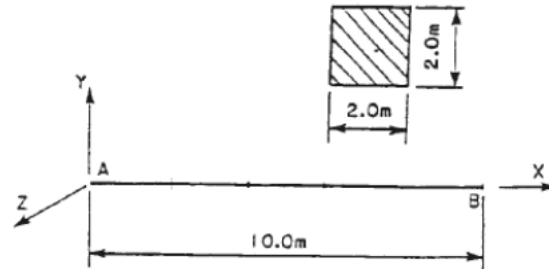
## Problem Description:

Calculation of natural frequencies (Hz) of the first 8 modes of a simply supported beam

From the NAFEMS reference:

### Geometry and Mesh

Exact beam: 5 elements



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV5. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

- Simply-supported beam model using BEAM elements.
- BEAM elements
- Repeated eigenvalues
- Lanczos method
- Shear deformation and rotary inertia
- Possibility of missing extensional modes when using iteration solution methods

## Boundary Condition:

$x = y = z = R_x = 0$  at A,  $y = z = 0$  at B

## Material Properties:

$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

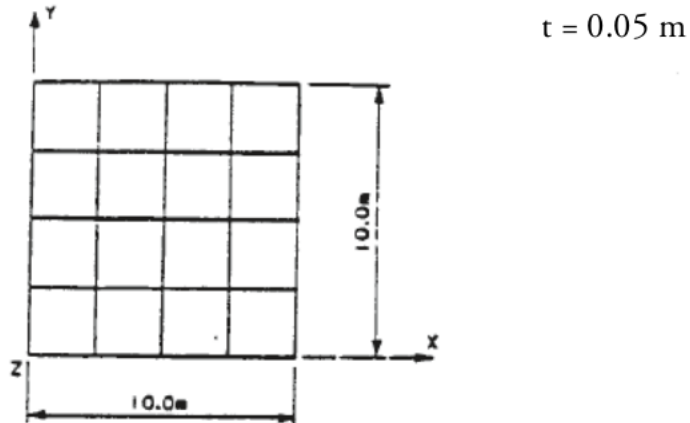
## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	42.649	42.867	43.111
2	42.649	42.867	43.111
3	77.542	77.204	77.204
4	125.0	124.487	124.487
5	148.31	148.505	149.393
6	148.31	148.505	149.393
7	233.1	224.056	224.056
8	284.55	273.695	269.578

# Free Thin Square Plate

## Problem Description:

Out of plane free vibration of a square plate with in-plane motion constrained. Find natural frequencies (Hz) of modes 4 to 10 (avoid modes 1-3 since they are rigid-body modes)



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV12. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

A simple flat plate model is created using quadrilateral elements and the out-of plane modes are calculated.

Rigid-body modes (3)

Repeated eigenvalues

Kinematically incomplete suppressions

## Boundary Condition:

$x = y = R_z = 0$  at all nodes

## Material Properties:

$E = 200 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

## RESULTS:

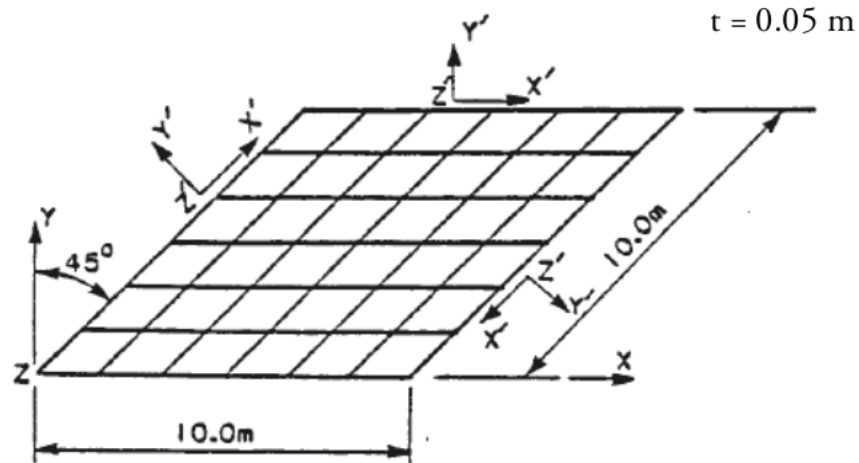
Mode	Theory	CEETRON Solve	MSC Nastran
4	1.622	1.586	1.532
5	2.36	2.254	2.356
6	2.922	2.83	2.831
7	4.233	3.97	4.122
8	4.233	3.97	4.122
9	7.416	7.025	7.363



# Clamped Thin Rhombic Plate

## Problem Description:

Solution to find the first 6 modes of a clamped flat plate, which is skewed 45 degrees.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV15. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

Flat quadrilateral elements modeling a clamped plate.  
Lanczos eigenvalue solution  
Distorted elements

## Boundary Condition:

$x = y = Rz = 0$  at all nodes  
 $Z' = Rx' = Ry' = 0$  along all 4 edges

## Material Properties:

$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

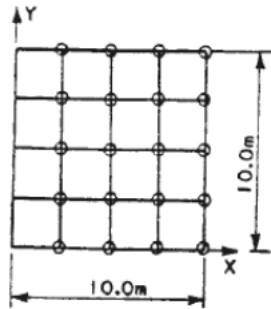
## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	7.938	8.046	7.832
2	12.835	13.468	12.851
3	17.941	19.159	18.078
4	19.133	19.489	18.585
5	24.009	26.248	24.31
6	27.922	29.912	27.644

# Cantilevered Thin Plate

## Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV16. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

A simple flat plate model created using quadrilateral elements.  
Normal modes calculation using quadrilateral elements

## Boundary Condition:

$x = y = z = R_y = 0$  along y-axis

## Material Properties:

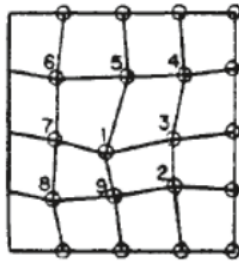
$E = 200 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.005
3	2.582	2.458	2.485
4	3.306	3.114	3.132
5	3.753	3.545	3.622
6	6.555	6.514	6.292

## Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



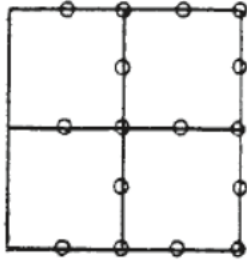
## Test 2

## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.419	0.415
2	1.029	1.03	1.007
3	2.582	2.509	2.509
4	3.306	3.132	3.164
5	3.753	3.854	3.664
6	6.555	6.383	6.327

### Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



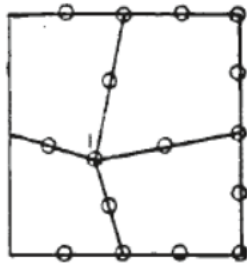
### Test 3

### RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.402	0.407
2	1.029	0.934	0.965
3	2.582	2.126	2.2
4	3.306	2.696	2.894
5	3.753	3.197	3.348
6	6.555	5.097	5.072

### Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



### Test 4

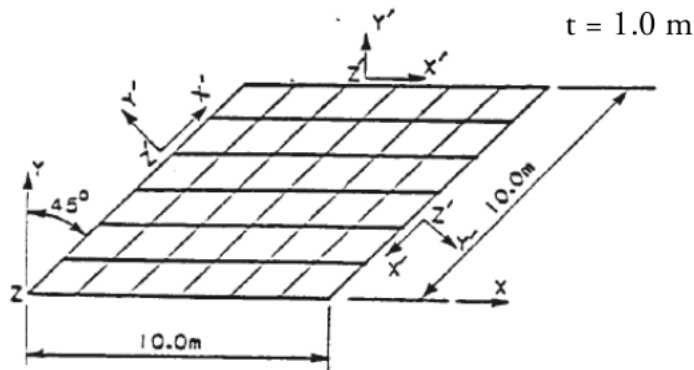
### RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.401	0.407
2	1.029	0.947	0.955
3	2.582	2.136	2.2
4	3.306	2.908	2.8
5	3.753	3.186	3.387
6	6.555	5.224	4.941

# Clamped Thick Rhombic Plate

## Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a clamped flat plate, which is skewed 45 degrees.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV22. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

Distorted Flat quadrilateral elements modeling a thick clamped plate.  
Lanczos Eigenvalue Solution  
Distorted elements

## Boundary Condition:

$x = y = Rz = 0$  at all nodes  
 $Z' = Rx' = Ry' = 0$  along all 4 edges

## Material Properties:

$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

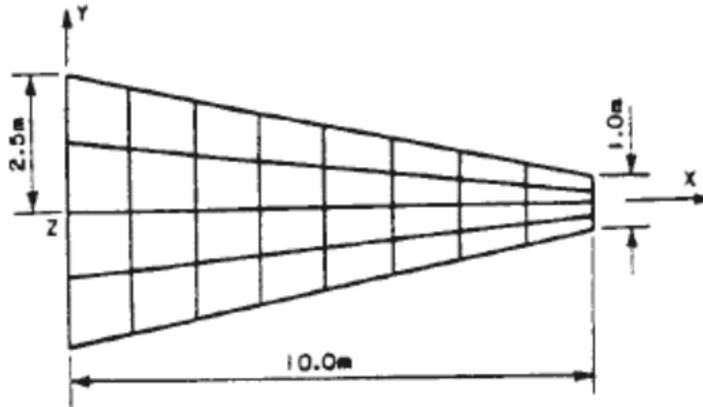
## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	133.95	136.248	131.22
2	201.41	211.734	200.37
3	265.81	282.71	262.03
4	282.74	287.623	273.59
5	334.45	360.539	327.01

# Cantilevered Tapered Membrane

## Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a tapered membrane plate with 4 node quadrilateral element.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV32. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

- Distorted Flat quadrilateral elements modeling a tapered membrane.
- Lanczos eigenvalue solution
- Shear behaviour
- Irregular mesh
- Symmetry

## Boundary Condition:

$z = 0$  at all nodes,  $x = y = 0$  along  $y -$  axis

## Material Properties:

$E = 200 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

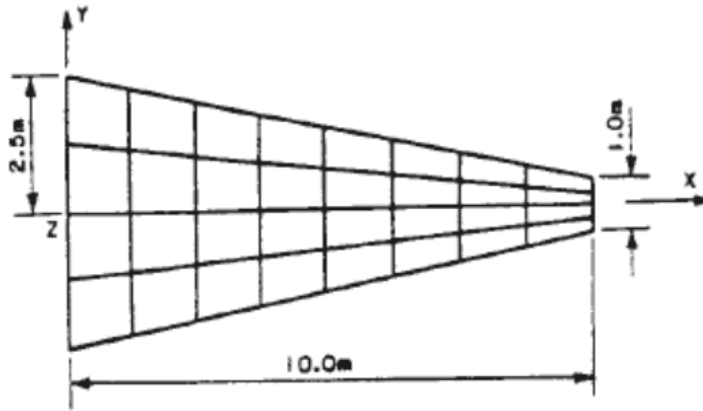
## RESULTS:



Mode	Theory	CEETRON Solve	MSC Nastran
1	44.623	44.631	44.52
2	130.03	129.833	129.55
3	162.7	162.618	162.56
4	246.05	244.648	244.13
5	379.9	375.251	374.46
6	391.44	389.853	389.6

## Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a tapered membrane plate with 8 node quadrilateral element.



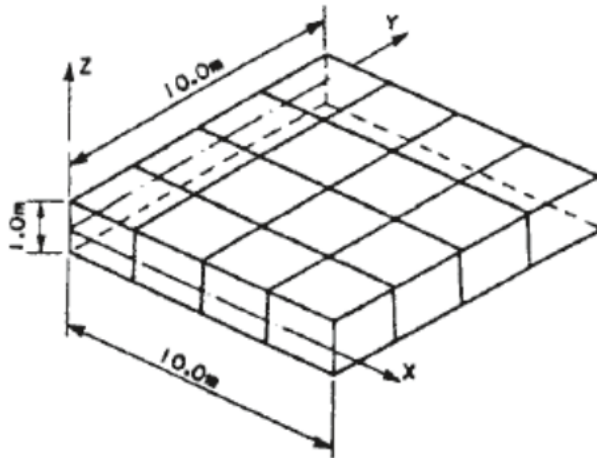
## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	44.623	44.342	44.54
2	130.03	128.383	129.71
3	162.7	162.476	162.66
4	246.05	241.25	245.14
5	379.9	369.405	377.87
6	391.44	389.487	390.92

# Simply Supported Solid Square Plate

## Problem Description:

Calculate the natural frequencies (Hz) of the first 10 modes of a plate which is supported in the Z-direction on its edges.



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV51. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

- Solid elements
- Rigid body modes (3 modes)
- Lanczos method
- Kinematically incomplete suppressions

## Boundary Condition:

$Z = 0$  along the 4 edges on the plane  $Z = -0.5\text{m}$

## Material Properties:

$E = 200 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

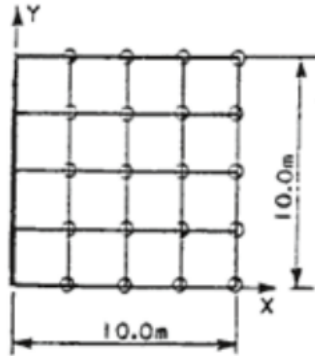
## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
4	45.897	44.502	43.81
5	109.44	107.948	105.24
6	109.44	107.948	105.24
7	167.89	161.437	156.26
8	193.59	185.59	193.97
9	206.19	185.59	193.52
10	206.19	193.162	193.52

# Cantilevered Thin Square Plate

## Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.  
Thickness = 0.05m



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV73. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

Rigid Body modes  
Lanczos method  
Effect of master degree of freedom selection on frequencies

## Boundary Condition:

$x = y = z = R_y = 0$  along y-axis

## Material Properties:

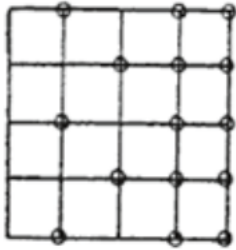
$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

## RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.005
3	2.582	2.458	2.485
4	3.306	3.114	3.15
5	3.753	3.545	3.622
6	6.555	6.514	6.292

### Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.  
Thickness = 0.05m



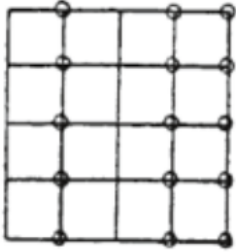
### Test 2

### RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.006
3	2.582	2.458	2.509
4	3.306	3.114	3.18
5	3.753	3.545	3.713
6	6.555	6.514	6.902

### Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.  
Thickness = 0.05m



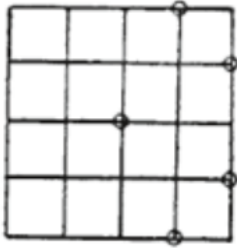
Test 3

### RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.007
3	2.582	2.458	2.563
4	3.306	3.114	3.196
5	3.753	3.545	3.828
6	6.555	6.514	6.879

### Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.  
Thickness = 0.05m



### Test 4

### RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.015
3	2.582	2.458	2.711
4	3.306	3.114	3.272
5	3.753	3.545	4.935
6	6.555	6.514	0.0