The Standard NAFEMS Benchmark Tests for CEETRON Solve

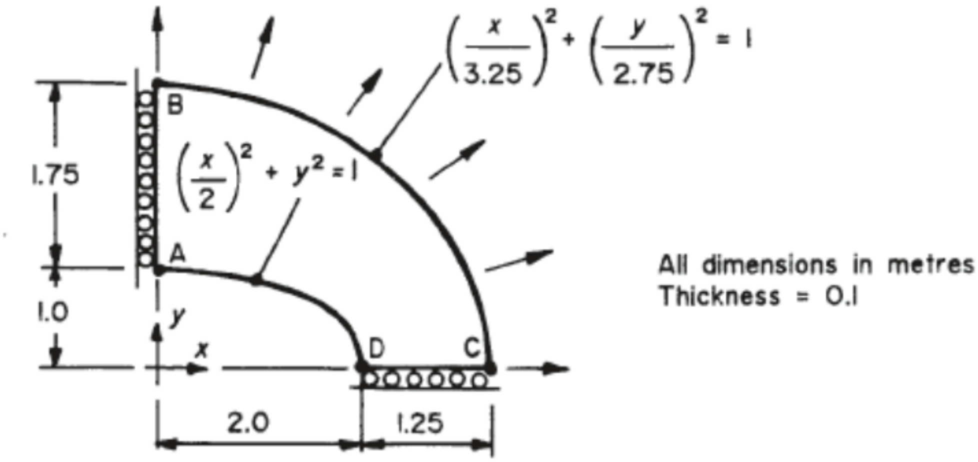
This document contains NAFEMS benchmark tests for CEETRON Solve version 2.8.0. Each test case provides a detailed overview of the problem and results compared between theoretical expectations, CEETRON Solve results, and MSC Nastran results.

Linear Elastic

Elliptic Membrane

Problem Description:

Find the tangential edge stress at D (refer below figure) of elliptical membrane which is uniformly loaded with outward pressure



Reference:

Test LE1 from NAFEMS Publication NNB, Rev. 3, NAFEMS Linear Benchmarks, 5 Oct 1990

Modeling Techniques Used:

Elliptical membrane is modeled using plane stress elements quadrilaterals.
Linear Static Analysis

Loading:

Uniform outward pressure of 10MPa at outer edge BC, Inner curved edge AD unloaded.

Boundary Condition:

Edge AB, symmetry about Y axis, e.g. zero x displacement
Edge CD, symmetry about X axis, e.g. zero y displacement.

Material Properties:

Isotropic, E = 210 x 10^3 Mpa and v = 0.3

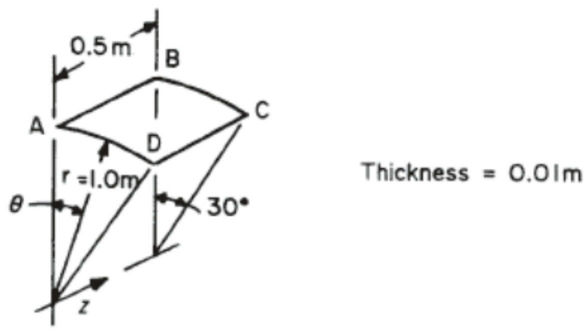
RESULTS:

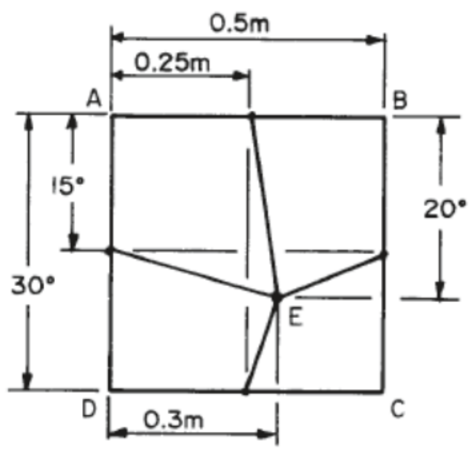
Theory	CEETRON Solve	MSC Nastran
92700000.0	94812752.0	91160000.0

Cylindrical Shell Patch Test

Problem Description:

Find the outer surface tangential stress at E (refer below figure) of cylindrical shell with uniform normal edge moment, on DC of 1.0 kNm/m





Reference:

Test LE2 from NAFEMS report TSBM, Publication NNB, Rev. 3, NAFEMS Linear Benchmarks, 5 Oct 1990

Modeling Techniques Used:

Cylindrical shell is modeled using plane stress quadrilateral elements.
Linear Static Analysis

Loading:

Uniform normal edge moment, on DC, of 1.0 kNm/m

Boundary Condition:

Edge AB, all translations and rotations zero
Edge AD, BC are symmetric about r-theta plane, e.g. Z translations and normal rotations all zero.

Material Properties:

Isotropic, E = 210 x 10^3 MPa and v = 0.3

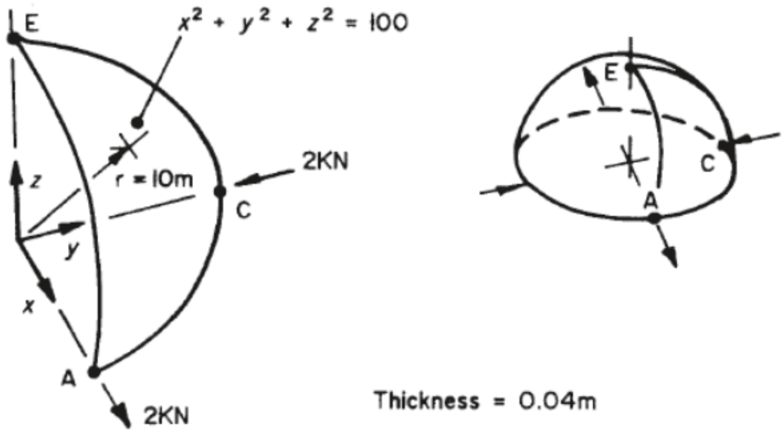
RESULTS:

Theory	CEETRON Solve	MSC Nastran
60000000.0	65775087.0	57300000.0

Hemisphere-Point Loads

Problem Description:

Find the displacement in x- direction of point A of hemisphere shown below



Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE3. Glasgow: NAFEMS, Rev. 3, 1990

Modeling Techniques Used:

Hemisphere shell is modeled using plane stress quadrilateral elements.
Linear Static Analysis

Loading:

Uniform normal pressure of 1 MPa on the upper surface of the plate

Boundary Condition:

Point E, zero z displacement
Edge AE, sylletry about zx plane; e.g zero y displacement, zero normal rotation
Edge CE, symmetry about yz plane, e.g zero x displacement, zero normal rotation

Material Properties:

Isotropic, E = 68.25 x 10^3 Mpa, v = 0.3

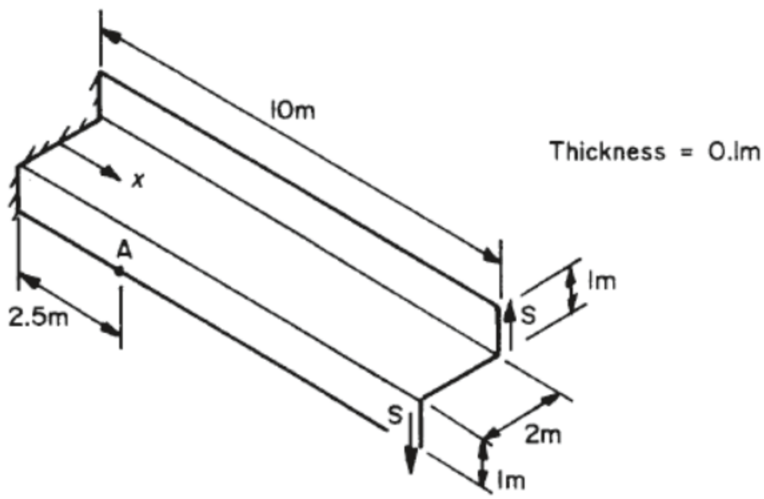
RESULTS:

Theory	CEETRON Solve	MSC Nastran
0.185	0.176	0.179

Z-section Cantilever

Problem Description:

A z-section cantilever beam is subjected to a torsion load. Find the axial stress (X-X) at mid-surface at Point A shown below



Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE5. Glasgow: NAFEMS, Rev. 3, 1990

Modeling Techniques Used:

Z shaped cantilever beam is modeled using plane stress quadrilateral elements
Linear Static Analysis

Loading:

Torque of 1.2 MNm applied at end x = 10 by two uniformly distributed. Edge shears, S = 0.6 at each flange.

Boundary Condition:

At edge x = 0, all displacements are zero.

Material Properties:

Isotropic, E = 210 x 10^3 MPa, v = 0.3

RESULTS:

Theory	CEETRON Solve	MSC Nastran
-108000000.0	-104137508.0	-103000000.0

Skew Plate Normal Pressure

Problem Description:

A skew plate is subjected to uniform normal pressure in the vertical z- direction. 4 node quadrilateral element is used. Find the maximum principal at plate center on bottom section.

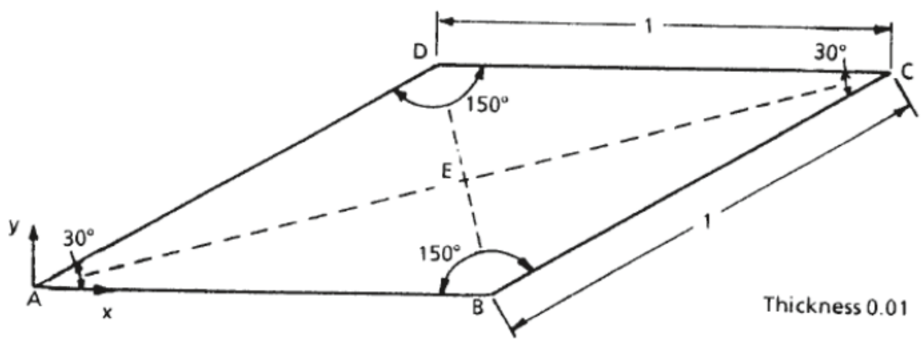


Figure 6.5-1 Skew Plate

Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE6. Glasgow: NAFEMS, Rev. 3, 1990.

Modeling Techniques Used:

Linear Static Analysis

Loading:

Normal pressure of -0.7KPa in vertical Z-direction

Boundary Condition:

Simple supports (no Z-displacement) for all edges AB, BC, CD, DA

Material Properties:

Isotropic, E = 210 x 10^3 MPa, v = 0.3

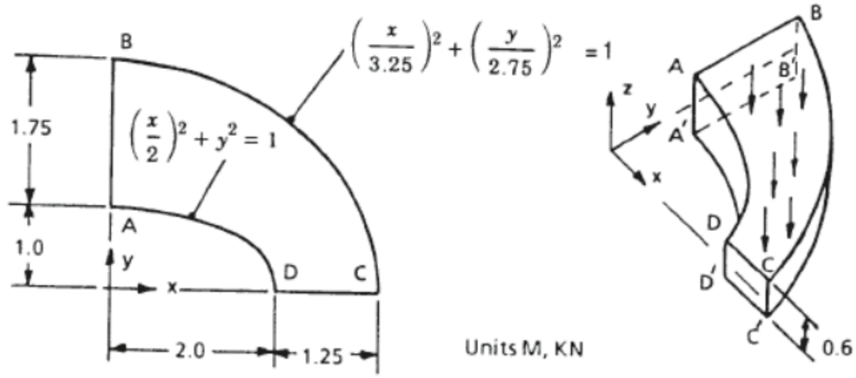
RESULTS:

Theory	CEETRON Solve	MSC Nastran
802000.0	703654.962	768000.0

Thick Plate Pressure

Problem Description:

A elliptical thick plate is subjected to uniform normal pressure on the upper surface of the plate. Find the direct Stress (Y-Y) at Point D shown below



Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE10. Glasgow: NAFEMS, Rev. 3, 1990

Modeling Techniques Used:

Thick elliptical plate is modeled using solid hexahedra elements. Linear Static Analysis

Loading:

Uniform normal pressure of 1 MPa on the upper surface of the Plate

Boundary Condition:

Face DCD'C' zero y-displacement
Face ABA'B' zero x- displacement
Face BCB'C' x and y displacements fixed, z displacements fixed along mid-plane

Material Properties:

Isotropic, E = 210 x 10^3 MPa, v = 0.3

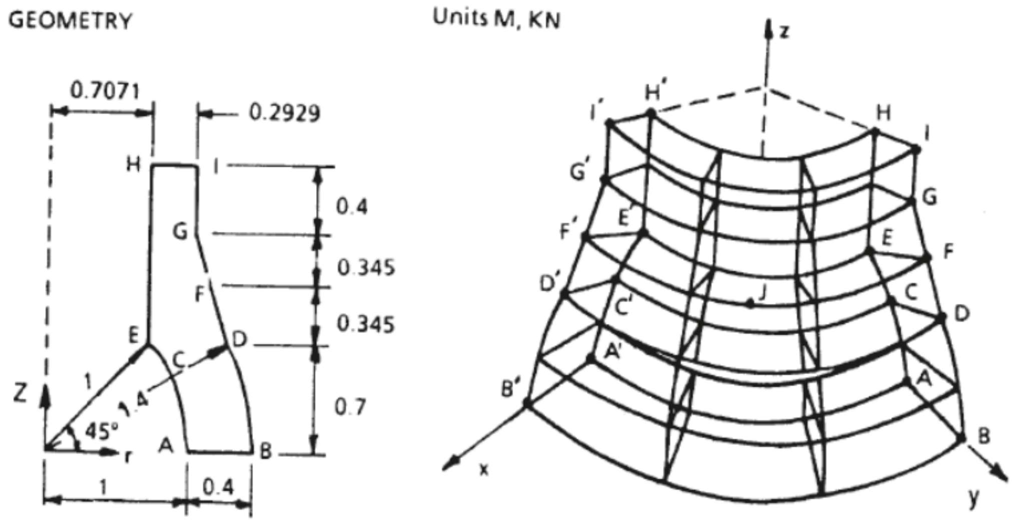
RESULTS:

Theory	CEETRON Solve	MSC Nastran
-5380000.0	-5642920.5	-6000000.0

Solid Cylinder/ Taper/ Sphere-Temperature

Problem Description:

A solid spherical taper cylinder is subjected temperature loading. Find the direct stress (Z-Z) at Point A



Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE11. Glasgow: NAFEMS, Rev. 3, 1990.

Modeling Techniques Used:

Solid cylinder is modeled using solid elements hexahedra. Linear Static Analysis

Loading:

Linear temperature gradient in the radial and axial direction T (°C) = $(x^2 + y^2)^{1/2} + z$

Boundary Condition:

Symmetry on x-z plane i.e., zero y- displacement,
Symmetry on y-z plane i.e., zero x- displacement,
Face on xy plane zero z- displacement,
Face HIH'I' zero z- displacement

Material Properties:

Isotropic, E = 210 x 10^3 MPa, v = 0.3, a = 0.00023 °C

RESULTS:

Theory	CEETRON Solve	MSC Nastran
-105000000.0	-93465800.0	-99477000.0

Free Vibration

Cantilever with Off-Centre Point Masses

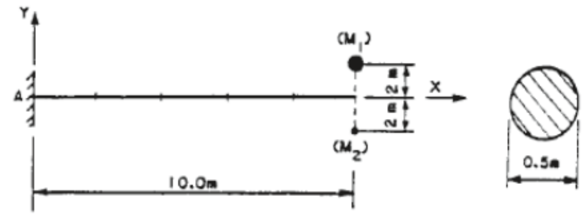
Problem Description:

Calculation of natural frequencies (Hz) of the first 6 modes of a cantilever beam with offset masses at the free end.

Geometry and Mesh

Exact beam: 5 elements along cantilever

M_1 = 10000 kg (along X, Y, Z)
M_2 = 1000 kg (along X, Y, Z)



Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C.Selected Benchmarks for Natural Frequency Analysis, Test No. FV4. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

Simple cantilever beam model.
BEAM elemnts.
Coupling between bending and torsion
Close eigenvalues
Inertial axis non-coincident with flexibility axis.

Boundary Condition:

x = y = z = Rx = Ry = Rz = 0 at A

Material Properties:

E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	1.723	1.723	1.723
2	1.727	1.727	1.727
3	7.413	7.424	7.45
4	9.972	9.972	9.975
5	18.155	18.162	18.205
6	26.957	26.973	27.001

Deep Simply-supported Beam

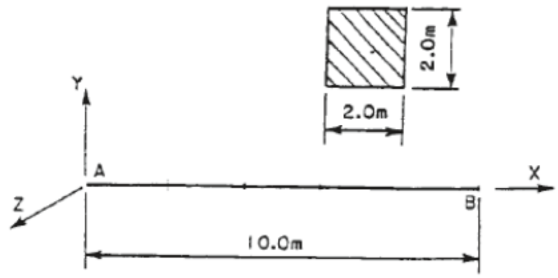
Problem Description:

Calculation of natural frequencies (Hz) of the first 8 modes of a simply supported beam

From the NAFEMS reference:

Geometry and Mesh

Exact beam: 5 elements



Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C.Selected Benchmarks for Natural Frequency Analysis, Test No. FV5. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

Simply-supported beam model using BEAM elements.
BEAM elements
Repeated eigenvalues
Lanczos method
Shear deformation and rotary inertia
Possibility of missing extensional modes when using iteration solution methods

Boundary Condition:

x = y = z = Rx = 0 at A, y = z = 0 at B

Material Properties:

E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

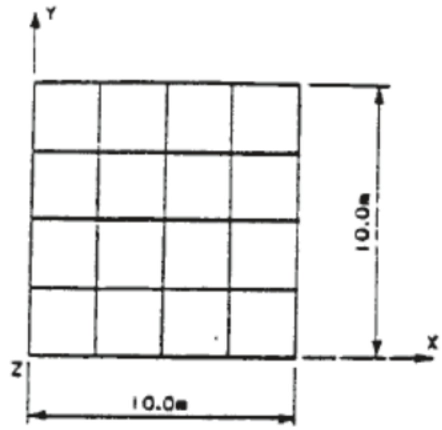
RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	42.649	42.867	43.111
2	42.649	42.867	43.111
3	77.542	77.204	77.204
4	125.0	124.487	124.487
5	148.31	148.505	149.393
6	148.31	148.505	149.393
7	233.1	224.056	224.056
8	284.55	273.695	269.578

Free Thin Square Plate

Problem Description:

Out of plane free vibration of a square plate with in-plane motion constrained. Find natural frequencies (Hz) of modes 4 to 10 (avoid modes 1-3 since they are rigid-body modes)



t = 0.05 m

Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV12. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

A simple flat plate model is created using quadrilateral elements and the out-of plane modes are calculated.
Rigid-body modes (3)
Repeated eigenvalues
Kinematically incomplete suppressions

Boundary Condition:

x = y = Rz = 0 at all nodes

Material Properties:

E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

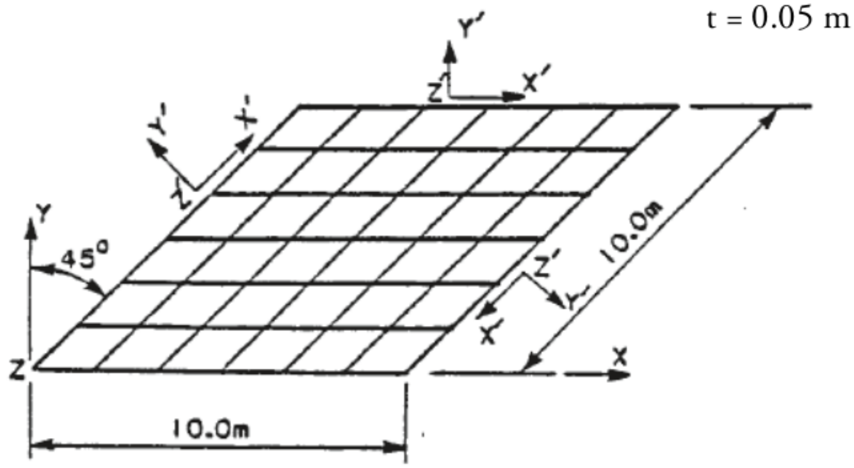
RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
4	1.622	1.586	1.532
5	2.36	2.254	2.356
6	2.922	2.83	2.831
7	4.233	3.97	4.122
8	4.233	3.97	4.122
9	7.416	7.025	7.363

Clamped Thin Rhombic Plate

Problem Description:

Solution to find the first 6 modes of a clamped flat plate, which is skewed 45 degrees.



Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV15. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

Flat quadrilateral elements modeling a clamped plate.
Lanczos eigenvalue solution
Distorted elements

Boundary Condition:

x = y = Rz = 0 at all nodes
Z' = Rx' = Ry' = 0 along all 4 edges

Material Properties:

E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

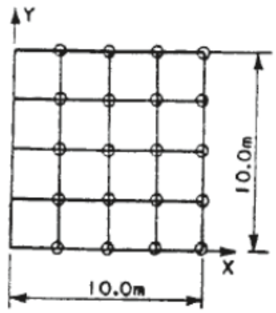
RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	7.938	8.046	7.832
2	12.835	13.468	12.851
3	17.941	19.159	18.078
4	19.133	19.489	18.585
5	24.009	26.248	24.31
6	27.922	29.912	27.644

Cantilevered Thin Plate

Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV16. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

A simple flat plate model created using quadrilateral elements.
Normal modes calculation using quadrilateral elements

Boundary Condition:

x = y = z = Ry = 0 along y-axis

Material Properties:

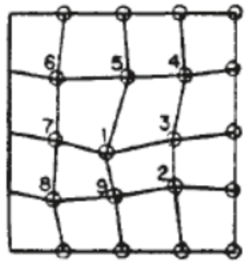
E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.005
3	2.582	2.458	2.485
4	3.306	3.114	3.132
5	3.753	3.545	3.622
6	6.555	6.514	6.292

Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



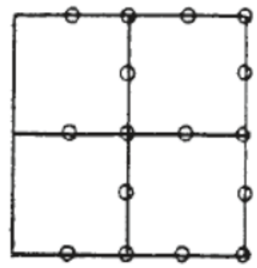
Test 2

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.419	0.415
2	1.029	1.03	1.007
3	2.582	2.509	2.509
4	3.306	3.132	3.164
5	3.753	3.854	3.664
6	6.555	6.383	6.327

Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



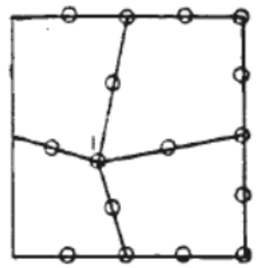
Test 3

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.402	0.407
2	1.029	0.934	0.965
3	2.582	2.126	2.2
4	3.306	2.696	2.894
5	3.753	3.197	3.348
6	6.555	5.097	5.072

Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



Test 4

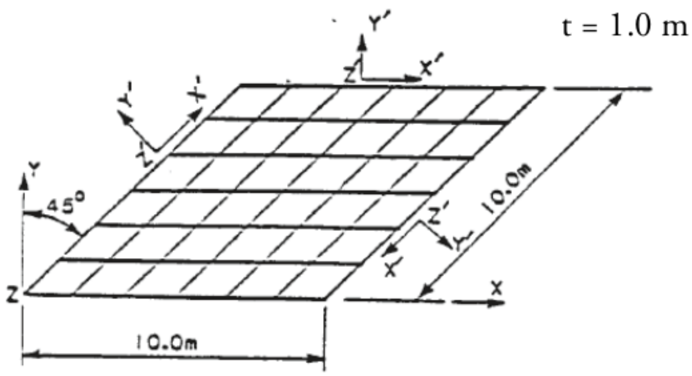
RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.401	0.407
2	1.029	0.947	0.955
3	2.582	2.136	2.2
4	3.306	2.908	2.8
5	3.753	3.186	3.387
6	6.555	5.224	4.941

Clamped Thick Rhombic Plate

Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a clamped flat plate, which is skewed 45 degrees.



Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV22. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

Distorted Flat quadrilateral elements modeling a thick clamped plate.
Lanczos Eigenvalue Solution
Distorted elements

Boundary Condition:

x = y = Rz = 0 at all nodes
Z' = Rx' = Ry' = 0 along all 4 edges

Material Properties:

E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

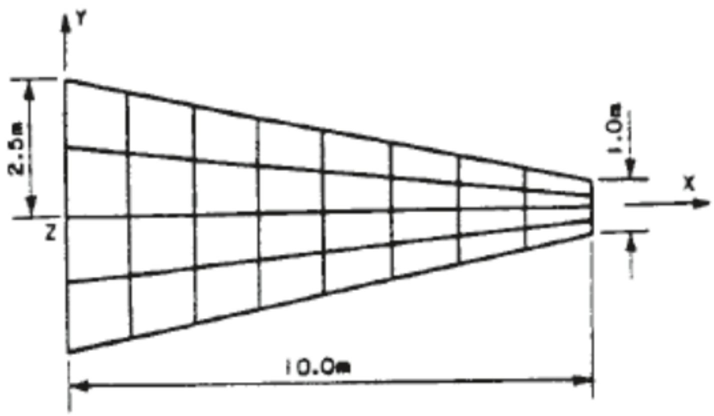
RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	133.95	136.248	131.22
2	201.41	211.734	200.37
3	265.81	282.71	262.03
4	282.74	287.623	273.59
5	334.45	360.539	327.01

Cantilevered Tapered Membrane

Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a tapered membrane plate with 4 node quadrilateral element.



Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV32. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

Distorted Flat quadrilateral elements modeling a tapered membrane.
Lanczos eigenvalue solution
Shear behaviour
Irregular mesh
Symmetry

Boundary Condition:

z = 0 at all nodes, x = y = 0 along y – axis

Material Properties:

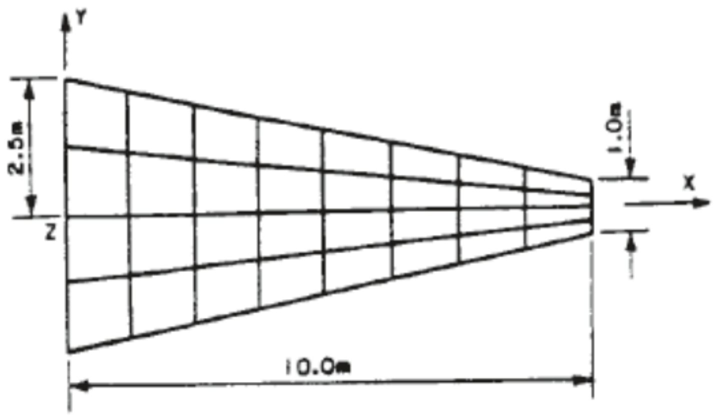
E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	44.623	44.631	44.52
2	130.03	129.833	129.55
3	162.7	162.618	162.56
4	246.05	244.648	244.13
5	379.9	375.251	374.46
6	391.44	389.853	389.6

Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a tapered membrane plate with 8 node quadrilateral element.



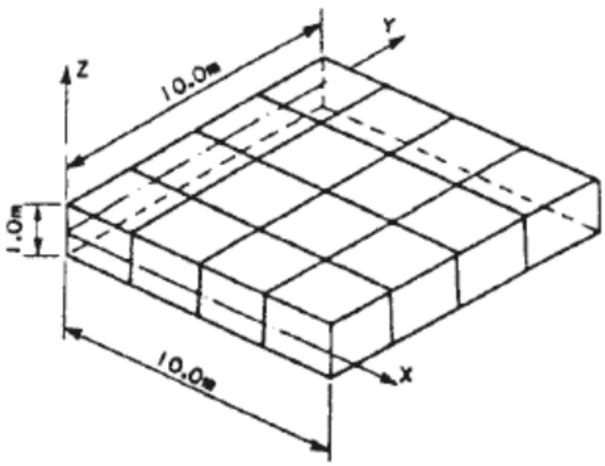
RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	44.623	44.342	44.54
2	130.03	128.383	129.71
3	162.7	162.476	162.66
4	246.05	241.25	245.14
5	379.9	369.405	377.87
6	391.44	389.487	390.92

Simply Supported Solid Square Plate

Problem Description:

Calculate the natural frequencies (Hz) of the first 10 modes of a plate which is supported in the Z-direction on its edges.



Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C.Selected Benchmarks for Natural Frequency Analysis, Test No. FV51. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

Solid elements
Rigid body modes (3 modes)
Lanczos method
Kinematically incomplete suppressions

Boundary Condition:

Z = 0 along the 4 edges on the plane Z = -0.5m

Material Properties:

E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

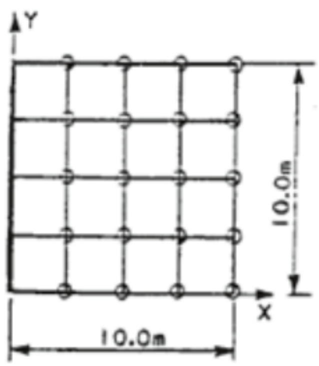
RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
4	45.897	44.502	43.81
5	109.44	107.948	105.24
6	109.44	107.948	105.24
7	167.89	161.437	156.26
8	193.59	185.59	193.97
9	206.19	185.59	193.52
10	206.19	193.162	193.52

Cantilevered Thin Square Plate

Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.
Thickness = 0.05m



Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C.Selected Benchmarks for Natural Frequency Analysis, Test No. FV73. Glasgow: NAFEMS, Nov., 1987.

Modeling Techniques Used:

Rigid Body modes
Lanczos method
Effect of master degree of freedom selection on frequencies

Boundary Condition:

x= y = z = Ry = 0 along y-axis

Material Properties:

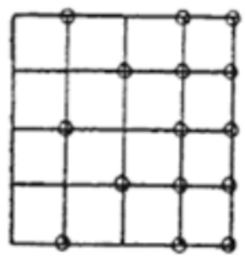
E = 200 x 10^3 MPa, v = 0.3, density = 8000 kg/m^3

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.005
3	2.582	2.458	2.485
4	3.306	3.114	3.15
5	3.753	3.545	3.622
6	6.555	6.514	6.292

Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.
Thickness = 0.05m



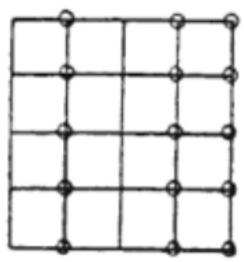
Test 2

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.006
3	2.582	2.458	2.509
4	3.306	3.114	3.18
5	3.753	3.545	3.713
6	6.555	6.514	6.902

Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.
Thickness = 0.05m



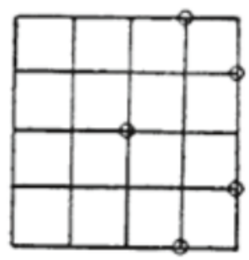
Test 3

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.007
3	2.582	2.458	2.563
4	3.306	3.114	3.196
5	3.753	3.545	3.828
6	6.555	6.514	6.879

Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis. Thickness = 0.05m



Test 4

RESULTS:

Mode	Theory	CEETRON Solve	MSC Nastran
1	0.421	0.415	0.415
2	1.029	0.999	1.015
3	2.582	2.458	2.711
4	3.306	3.114	3.272
5	3.753	3.545	4.935
6	6.555	6.514	0.0