

# The Standard NAFEMS Benchmark Tests for HOOPS Solve

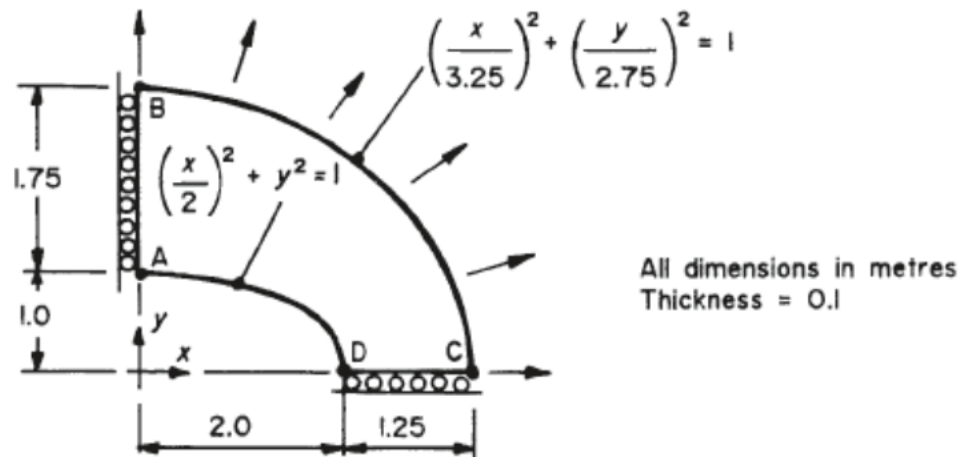
This document contains NAFEMS benchmark tests for HOOPS Solve version 2.15.0. Each test case provides a detailed overview of the problem and results compared between theoretical expectations, HOOPS Solve results, and MSC Nastran results.

# **Linear Elastic**

# Elliptic Membrane

## Problem Description:

Find the tangential edge stress at D (refer below figure) of elliptical membrane which is uniformly loaded with outward pressure



## Reference:

Test LE1 from NAFEMS Publication NNB, Rev. 3, NAFEMS Linear Benchmarks, 5 Oct 1990

## Modeling Techniques Used:

Elliptical membrane is modeled using plane stress elements quadrilaterals.  
Linear Static Analysis

## Loading:

Uniform outward pressure of 10MPa at outer edge BC, Inner curved edge AD unloaded.

## Boundary Condition:

Edge AB, symmetry about Y axis, e.g. zero x displacement  
Edge CD, symmetry about X axis, e.g. zero y displacement.

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  Mpa and  $\nu = 0.3$

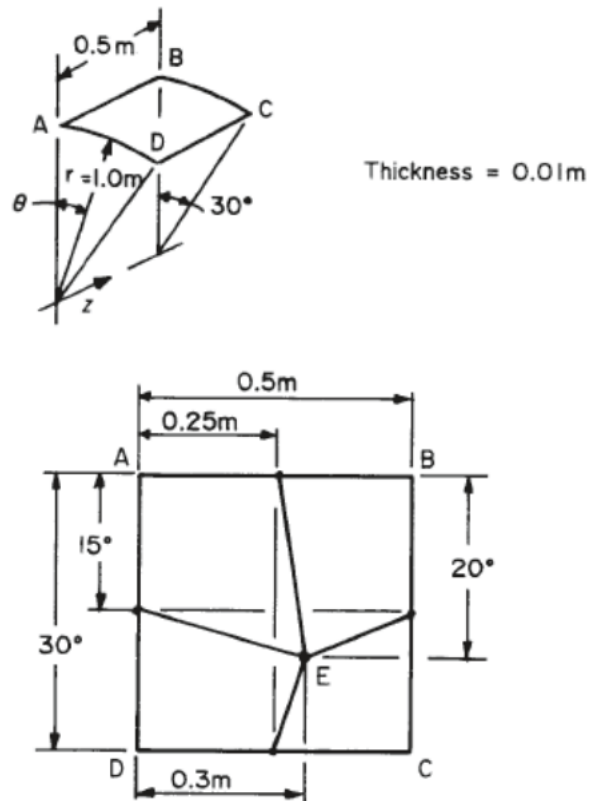
## RESULTS:

| Theory     | HOOPS Solve | MSC Nastran |
|------------|-------------|-------------|
| 92700000.0 | 94812752.0  | 91160000.0  |

# Cylindrical Shell Patch Test

## Problem Description:

Find the outer surface tangential stress at E (refer below figure) of cylindrical shell with uniform normal edge moment, on DC of  $1.0 \text{ kNm/m}$



## Reference:

Test LE2 from NAFEMS report TSBM, Publication NNB, Rev. 3, NAFEMS Linear Benchmarks, 5 Oct 1990

## Modeling Techniques Used:

Cylindrical shell is modeled using plane stress quadrilateral elements.  
Linear Static Analysis

## Loading:

Uniform normal edge moment, on DC, of  $1.0 \text{ kNm/m}$

## Boundary Condition:

Edge AB, all translations and rotations zero  
Edge AD, BC are symmetric about r-theta plane, e.g. Z translations and normal rotations all zero.

## Material Properties:

Isotropic,  $E = 210 \times 10^3 \text{ MPa}$  and  $\nu = 0.3$

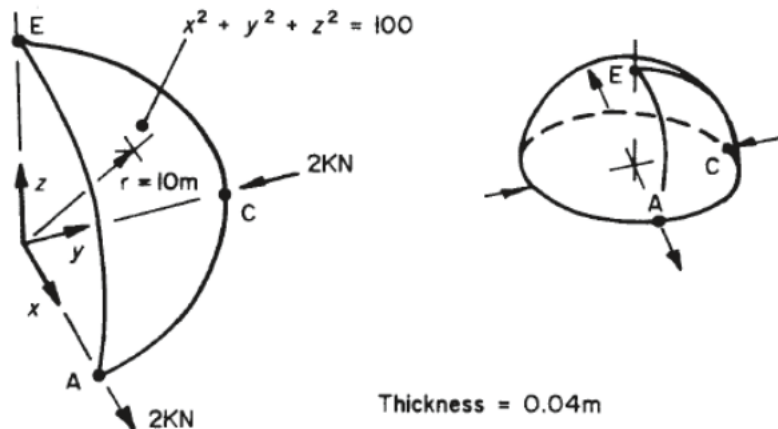
**RESULTS:**

| Theory     | HOOPS Solve | MSC Nastran |
|------------|-------------|-------------|
| 60000000.0 | 65775087.0  | 57300000.0  |

# Hemisphere-Point Loads

## Problem Description:

Find the displacement in x- direction of point A of hemisphere shown below



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE3. Glasgow: NAFEMS, Rev. 3, 1990

## Modeling Techniques Used:

Hemisphere shell is modeled using plane stress quadrilateral elements.  
Linear Static Analysis

## Loading:

Uniform normal pressure of  $1\text{ MPa}$  on the upper surface of the plate

## Boundary Condition:

Point E, zero  $z$  displacement  
Edge AE, symmetry about  $zx$  plane; e.g zero  $y$  displacement, zero normal rotation  
Edge CE, symmetry about  $yz$  plane, e.g zero  $x$  displacement, zero normal rotation

## Material Properties:

Isotropic,  $E = 68.25 \times 10^3\text{ Mpa}$ ,  $\nu = 0.3$

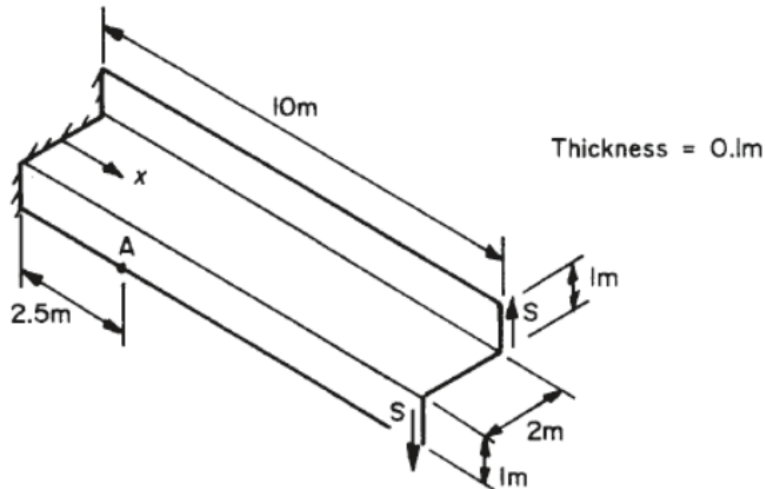
## RESULTS:

| Theory | HOOPS Solve | MSC Nastran |
|--------|-------------|-------------|
| 0.185  | 0.176       | 0.179       |

# Z-section Cantilever

## Problem Description:

A z-section cantilever beam is subjected to a torsion load. Find the axial stress (X-X) at mid-surface at Point A shown below



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE5. Glasgow: NAFEMS, Rev. 3, 1990

## Modeling Techniques Used:

Z shaped cantilever beam is modeled using plane stress quadrilateral elements  
Linear Static Analysis

## Loading:

Torque of 1.2 MNm applied at end  $x = 10$  by two uniformly distributed. Edge shears,  $S = 0.6$  at each flange.

## Boundary Condition:

At edge  $x = 0$ , all displacements are zero.

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  MPa,  $\nu = 0.3$

## RESULTS:

| Theory       | HOOPS Solve  | MSC Nastran  |
|--------------|--------------|--------------|
| -108000000.0 | -104137508.0 | -103000000.0 |

# Skew Plate Normal Pressure

## Problem Description:

A skew plate is subjected to uniform normal pressure in the vertical z- direction. 4 node quadrilateral element is used. Find the maximum principal at plate center on bottom section.

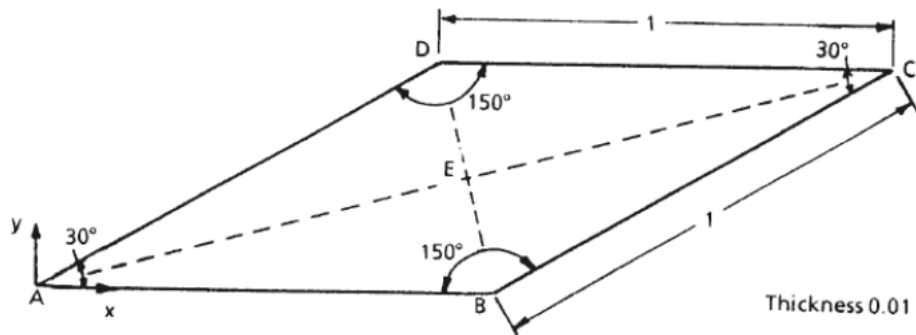


Figure 6.5-1 Skew Plate

## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE6. Glasgow: NAFEMS, Rev. 3, 1990.

## Modeling Techniques Used:

Linear Static Analysis

## Loading:

Normal pressure of -0.7KPa in vertical Z-direction

## Boundary Condition:

Simple supports (no Z-displacement) for all edges AB, BC, CD, DA

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  MPa,  $\nu = 0.3$

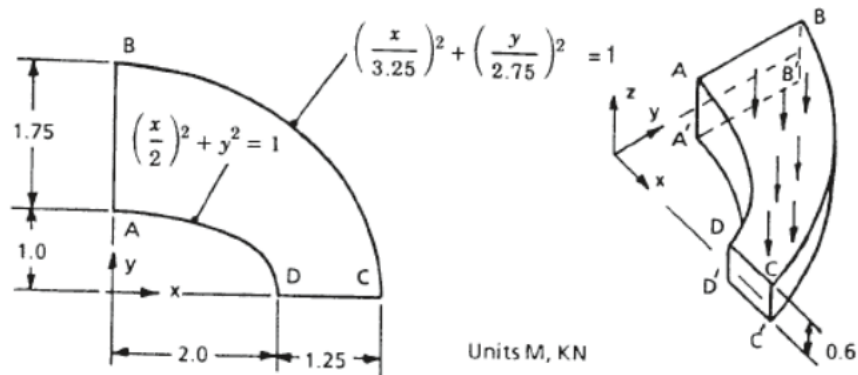
## RESULTS:

| Theory   | HOOPS Solve | MSC Nastran |
|----------|-------------|-------------|
| 802000.0 | 703654.962  | 768000.0    |

# Thick Plate Pressure

## Problem Description:

A elliptical thick plate is subjected to uniform normal pressure on the upper surface of the plate. Find the direct Stress (Y-Y) at Point D shown below



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE10. Glasgow: NAFEMS, Rev. 3, 1990

## Modeling Techniques Used:

Thick elliptical plate is modeled using solid hexahedra elements. Linear Static Analysis

## Loading:

Uniform normal pressure of 1 MPa on the upper surface of the Plate

## Boundary Condition:

Face DCD'C' zero y-displacement

Face ABA'B' zero x- displacement

Face BCB'C' x and y displacements fixed, z displacements fixed along mid-plane

## Material Properties:

Isotropic,  $E = 210 \times 10^3$  MPa,  $\nu = 0.3$

## RESULTS:

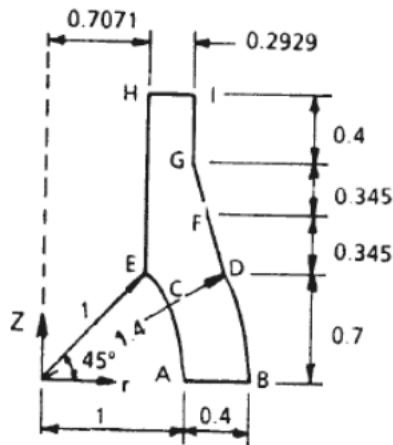
| Theory     | HOOPS Solve | MSC Nastran |
|------------|-------------|-------------|
| -5380000.0 | -5642920.5  | -6000000.0  |

# Solid Cylinder/ Taper/ Sphere-Temperature

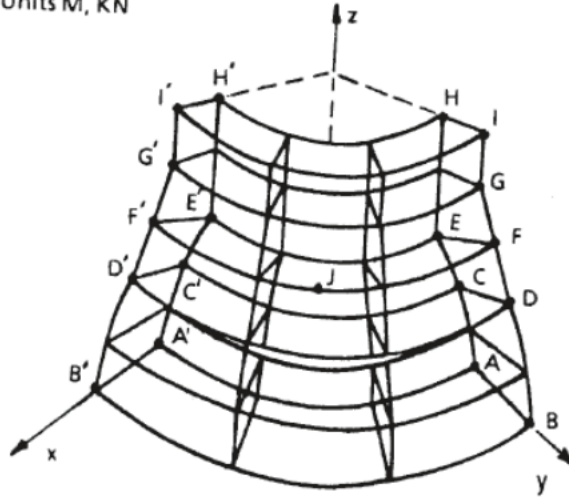
## Problem Description:

A solid spherical taper cylinder is subjected temperature loading. Find the direct stress (Z-Z) at Point A

GEOMETRY



Units M, KN



## Reference:

NAFEMS Finite Element Methods & Standards, The Standard NAFEMS Benchmarks, Test No. LE11. Glasgow: NAFEMS, Rev. 3, 1990.

## Modeling Techniques Used:

Solid cylinder is modeled using solid elements hexahedra. Linear Static Analysis

## Loading:

Linear temperature gradient in the radial and axial direction  $T (^{\circ}\text{C}) = (x^2 + y^2)^{1/2} + z$

## Boundary Condition:

Symmetry on x-z plane i.e., zero y- displacement,  
Symmetry on y-z plane i.e., zero x- displacement,  
Face on xy plane zero z- displacement,  
Face HH'I' zero z- displacement

## Material Properties:

Isotropic,  $E = 210 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ ,  $\alpha = 0.00023 ^{\circ}\text{C}$

## RESULTS:

| Theory       | HOOPS Solve | MSC Nastran |
|--------------|-------------|-------------|
| -105000000.0 | -93465800.0 | -99477000.0 |

# Free Vibration

# Cantilever with Off-Centre Point Masses

## Problem Description:

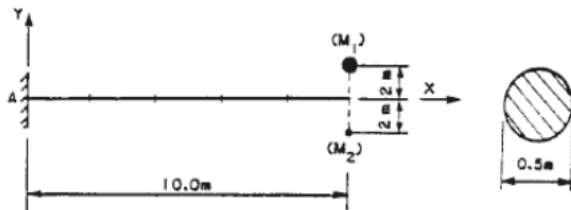
Calculation of natural frequencies (Hz) of the first 6 modes of a cantilever beam with offset masses at the free end.

### Geometry and Mesh

Exact beam: 5 elements along cantilever

$$M_1 = 10000 \text{ kg} \quad (\text{along X, Y, Z})$$

$$M_2 = 1000 \text{ kg} \quad (\text{along X, Y, Z})$$



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV4. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

- Simple cantilever beam model.
- BEAM elemnts.
- Coupling between bending and torsion
- Close eigenvalues
- Inertial axis non-coincident with flexibility axis.

## Boundary Condition:

$$x = y = z = R_x = R_y = R_z = 0 \text{ at A}$$

## Material Properties:

$$E = 200 \times 10^3 \text{ MPa}, \nu = 0.3, \text{ density} = 8000 \text{ kg/m}^3$$

## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 1.723  | 1.723       | 1.723       |
| 2    | 1.727  | 1.727       | 1.727       |
| 3    | 7.413  | 7.424       | 7.45        |
| 4    | 9.972  | 9.972       | 9.975       |
| 5    | 18.155 | 18.162      | 18.205      |
| 6    | 26.957 | 26.973      | 27.001      |

# Deep Simply-supported Beam

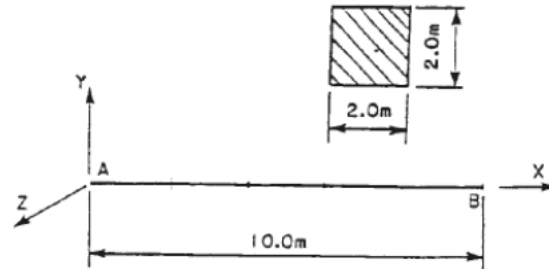
## Problem Description:

Calculation of natural frequencies (Hz) of the first 8 modes of a simply supported beam

From the NAFEMS reference:

### Geometry and Mesh

Exact beam: 5 elements



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV5. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

- Simply-supported beam model using BEAM elements.
- BEAM elements
- Repeated eigenvalues
- Lanczos method
- Shear deformation and rotary inertia
- Possibility of missing extensional modes when using iteration solution methods

## Boundary Condition:

$x = y = z = R_x = 0$  at A,  $y = z = 0$  at B

## Material Properties:

$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

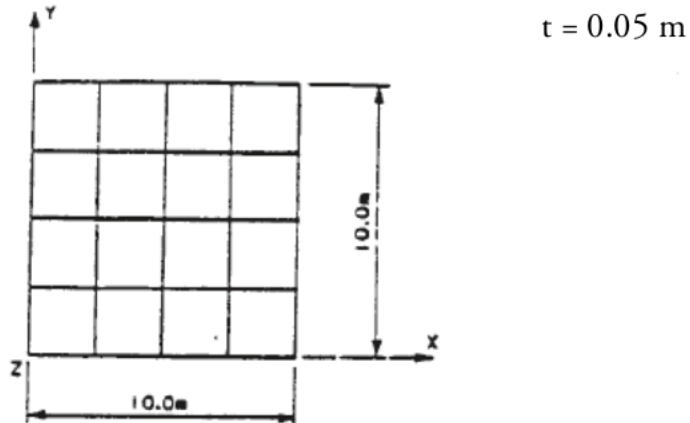
## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 42.649 | 42.867      | 43.111      |
| 2    | 42.649 | 42.867      | 43.111      |
| 3    | 77.542 | 77.204      | 77.204      |
| 4    | 125.0  | 124.487     | 124.487     |
| 5    | 148.31 | 148.505     | 149.393     |
| 6    | 148.31 | 148.505     | 149.393     |
| 7    | 233.1  | 224.056     | 224.056     |
| 8    | 284.55 | 273.695     | 269.578     |

# Free Thin Square Plate

## Problem Description:

Out of plane free vibration of a square plate with in-plane motion constrained. Find natural frequencies (Hz) of modes 4 to 10 (avoid modes 1-3 since they are rigid-body modes)



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV12. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

A simple flat plate model is created using quadrilateral elements and the out-of plane modes are calculated.

Rigid-body modes (3)

Repeated eigenvalues

Kinematically incomplete suppressions

## Boundary Condition:

$x = y = R_z = 0$  at all nodes

## Material Properties:

$E = 200 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

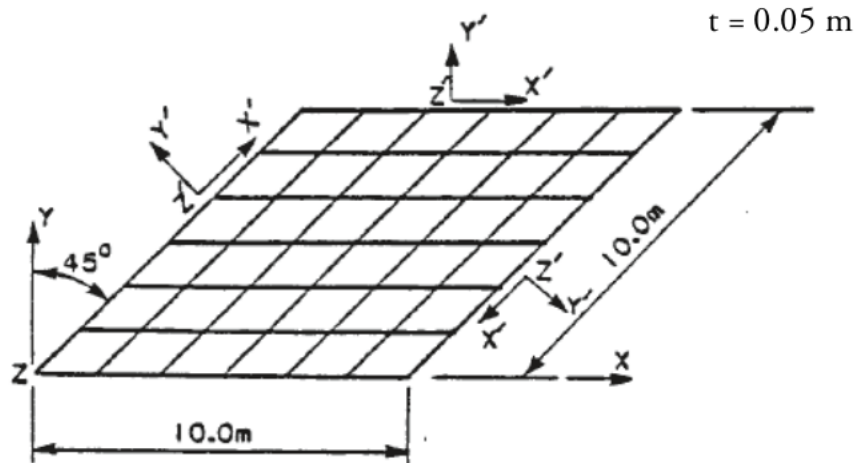
## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 4    | 1.622  | 1.586       | 1.532       |
| 5    | 2.36   | 2.254       | 2.356       |
| 6    | 2.922  | 2.83        | 2.831       |
| 7    | 4.233  | 3.97        | 4.122       |
| 8    | 4.233  | 3.97        | 4.122       |
| 9    | 7.416  | 7.025       | 7.363       |

# Clamped Thin Rhombic Plate

## Problem Description:

Solution to find the first 6 modes of a clamped flat plate, which is skewed 45 degrees.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV15. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

Flat quadrilateral elements modeling a clamped plate.  
Lanczos eigenvalue solution  
Distorted elements

## Boundary Condition:

$x = y = Rz = 0$  at all nodes  
 $Z' = Rx' = Ry' = 0$  along all 4 edges

## Material Properties:

$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

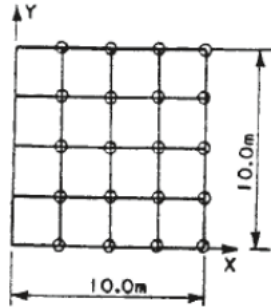
## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 7.938  | 8.046       | 7.832       |
| 2    | 12.835 | 13.468      | 12.851      |
| 3    | 17.941 | 19.159      | 18.078      |
| 4    | 19.133 | 19.489      | 18.585      |
| 5    | 24.009 | 26.248      | 24.31       |
| 6    | 27.922 | 29.912      | 27.644      |

# Cantilevered Thin Plate

## Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV16. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

A simple flat plate model created using quadrilateral elements.  
Normal modes calculation using quadrilateral elements

## Boundary Condition:

$x = y = z = R_y = 0$  along y-axis

## Material Properties:

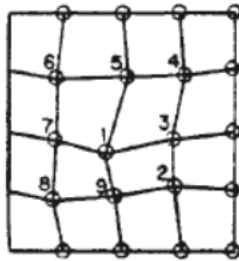
$E = 200 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.415       | 0.415       |
| 2    | 1.029  | 0.999       | 1.005       |
| 3    | 2.582  | 2.458       | 2.485       |
| 4    | 3.306  | 3.114       | 3.132       |
| 5    | 3.753  | 3.545       | 3.622       |
| 6    | 6.555  | 6.514       | 6.292       |

## Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



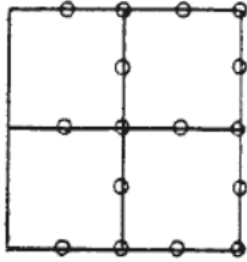
## Test 2

## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.419       | 0.415       |
| 2    | 1.029  | 1.03        | 1.007       |
| 3    | 2.582  | 2.509       | 2.509       |
| 4    | 3.306  | 3.132       | 3.164       |
| 5    | 3.753  | 3.854       | 3.664       |
| 6    | 6.555  | 6.383       | 6.327       |

### Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



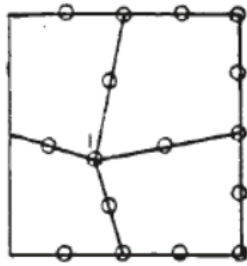
### Test 3

### RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.402       | 0.407       |
| 2    | 1.029  | 0.934       | 0.965       |
| 3    | 2.582  | 2.126       | 2.2         |
| 4    | 3.306  | 2.696       | 2.894       |
| 5    | 3.753  | 3.197       | 3.348       |
| 6    | 6.555  | 5.097       | 5.072       |

### Problem Description:

Normal modes of a cantilevered thin plate modeled using 8 nodes quadratic shell elements.



### Test 4

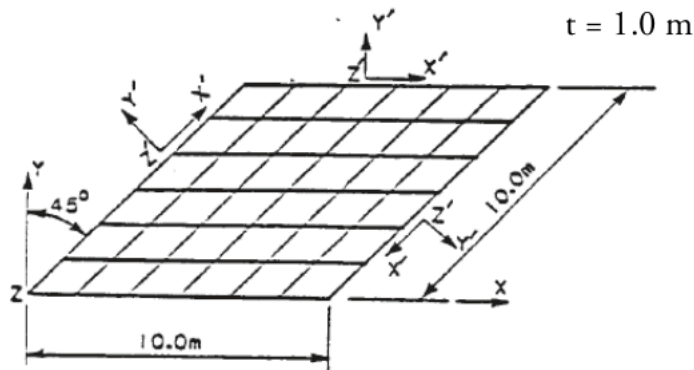
### RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.401       | 0.407       |
| 2    | 1.029  | 0.947       | 0.955       |
| 3    | 2.582  | 2.136       | 2.2         |
| 4    | 3.306  | 2.908       | 2.8         |
| 5    | 3.753  | 3.186       | 3.387       |
| 6    | 6.555  | 5.224       | 4.941       |

# Clamped Thick Rhombic Plate

## Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a clamped flat plate, which is skewed 45 degrees.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV22. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

Distorted Flat quadrilateral elements modeling a thick clamped plate.  
Lanczos Eigenvalue Solution  
Distorted elements

## Boundary Condition:

$x = y = Rz = 0$  at all nodes  
 $Z' = Rx' = Ry' = 0$  along all 4 edges

## Material Properties:

$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

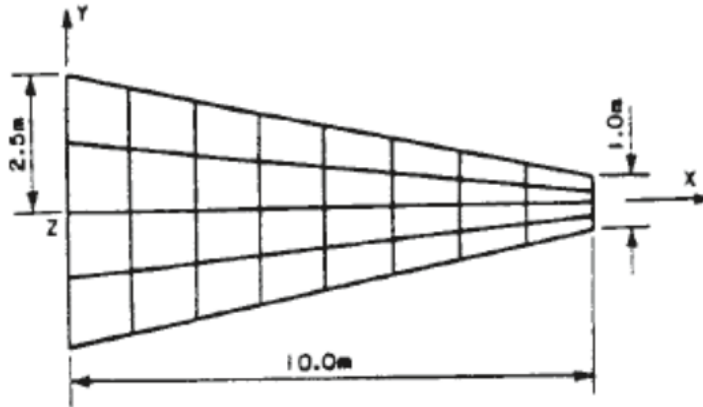
## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 133.95 | 136.248     | 131.22      |
| 2    | 201.41 | 211.734     | 200.37      |
| 3    | 265.81 | 282.71      | 262.03      |
| 4    | 282.74 | 287.623     | 273.59      |
| 5    | 334.45 | 360.539     | 327.01      |

# Cantilevered Tapered Membrane

## Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a tapered membrane plate with 4 node quadrilateral element.



## Reference:

NAFEMS Finite Element Methods & Standards, Abbassian, F., Dawswell, D. J., and Knowles, N. C., Selected Benchmarks for Natural Frequency Analysis, Test No. FV32. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

- Distorted Flat quadrilateral elements modeling a tapered membrane.
- Lanczos eigenvalue solution
- Shear behaviour
- Irregular mesh
- Symmetry

## Boundary Condition:

$z = 0$  at all nodes,  $x = y = 0$  along  $y -$  axis

## Material Properties:

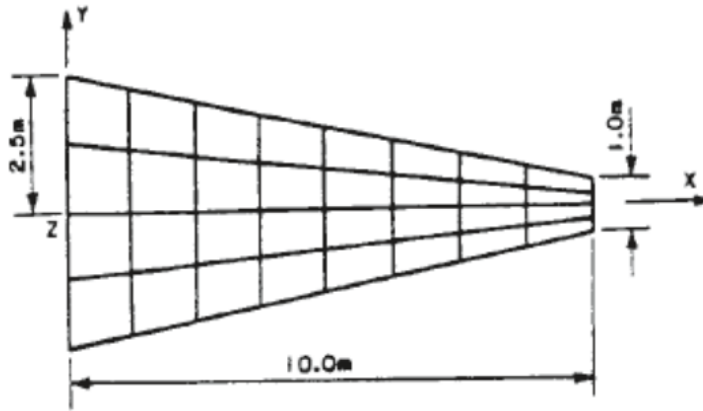
$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 44.623 | 44.631      | 44.52       |
| 2    | 130.03 | 129.833     | 129.55      |
| 3    | 162.7  | 162.618     | 162.56      |
| 4    | 246.05 | 244.648     | 244.13      |
| 5    | 379.9  | 375.251     | 374.46      |
| 6    | 391.44 | 389.853     | 389.6       |

## Problem Description:

Solution to find the natural frequencies (Hz) of the first 6 modes of a tapered membrane plate with 8 node quadrilateral element.



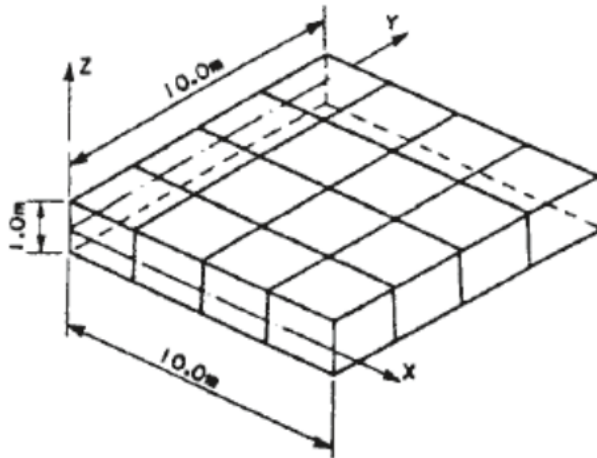
## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 44.623 | 44.342      | 44.54       |
| 2    | 130.03 | 128.383     | 129.71      |
| 3    | 162.7  | 162.476     | 162.66      |
| 4    | 246.05 | 241.25      | 245.14      |
| 5    | 379.9  | 369.405     | 377.87      |
| 6    | 391.44 | 389.487     | 390.92      |

# Simply Supported Solid Square Plate

## Problem Description:

Calculate the natural frequencies (Hz) of the first 10 modes of a plate which is supported in the Z-direction on its edges.



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV51. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

- Solid elements
- Rigid body modes (3 modes)
- Lanczos method
- Kinematically incomplete suppressions

## Boundary Condition:

$Z = 0$  along the 4 edges on the plane  $Z = -0.5\text{m}$

## Material Properties:

$E = 200 \times 10^3 \text{ MPa}$ ,  $\nu = 0.3$ , density =  $8000 \text{ kg/m}^3$

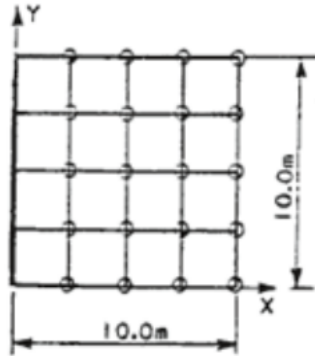
## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 4    | 45.897 | 44.502      | 43.81       |
| 5    | 109.44 | 107.948     | 105.24      |
| 6    | 109.44 | 107.948     | 105.24      |
| 7    | 167.89 | 161.437     | 156.26      |
| 8    | 193.59 | 185.59      | 193.97      |
| 9    | 206.19 | 185.59      | 193.52      |
| 10   | 206.19 | 193.162     | 193.52      |

# Cantilevered Thin Square Plate

## Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis. Thickness = 0.05m



## Reference:

NAFEMS Finite Element Methods & Standards. Abbassian, F., Dawswell, D. J., and Knowles, N. C. Selected Benchmarks for Natural Frequency Analysis, Test No. FV73. Glasgow: NAFEMS, Nov., 1987.

## Modeling Techniques Used:

Rigid Body modes  
Lanczos method  
Effect of master degree of freedom selection on frequencies

## Boundary Condition:

$x = y = z = R_y = 0$  along y-axis

## Material Properties:

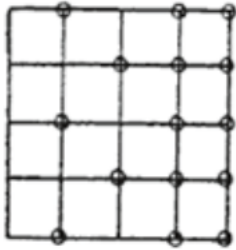
$E = 200 \times 10^3$  MPa,  $\nu = 0.3$ , density = 8000 kg/m<sup>3</sup>

## RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.415       | 0.415       |
| 2    | 1.029  | 0.999       | 1.005       |
| 3    | 2.582  | 2.458       | 2.485       |
| 4    | 3.306  | 3.114       | 3.15        |
| 5    | 3.753  | 3.545       | 3.622       |
| 6    | 6.555  | 6.514       | 6.292       |

### Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.  
Thickness = 0.05m



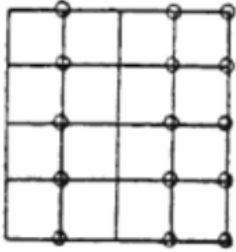
### Test 2

### RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.415       | 0.415       |
| 2    | 1.029  | 0.999       | 1.006       |
| 3    | 2.582  | 2.458       | 2.509       |
| 4    | 3.306  | 3.114       | 3.18        |
| 5    | 3.753  | 3.545       | 3.713       |
| 6    | 6.555  | 6.514       | 6.902       |

### Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.  
Thickness = 0.05m



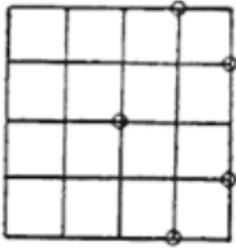
Test 3

### RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.415       | 0.415       |
| 2    | 1.029  | 0.999       | 1.007       |
| 3    | 2.582  | 2.458       | 2.563       |
| 4    | 3.306  | 3.114       | 3.196       |
| 5    | 3.753  | 3.545       | 3.828       |
| 6    | 6.555  | 6.514       | 6.879       |

### Problem Description:

Calculate the natural frequencies (Hz) of the first 6 modes of a plate which is simply supported along the Y-axis.  
Thickness = 0.05m



Test 4

### RESULTS:

| Mode | Theory | HOOPS Solve | MSC Nastran |
|------|--------|-------------|-------------|
| 1    | 0.421  | 0.415       | 0.415       |
| 2    | 1.029  | 0.999       | 1.015       |
| 3    | 2.582  | 2.458       | 2.711       |
| 4    | 3.306  | 3.114       | 3.272       |
| 5    | 3.753  | 3.545       | 4.935       |
| 6    | 6.555  | 6.514       | 0.0         |